B. Sc- part-I (Sub) Set theory 04-07-2020.

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Q.1. De Morgan's Laws

If A and B are two subsets of the universal set, then

(I) $(A \cup B)' = A' \cap B'$ (II) $(A \cap B)' = A' \cup B'$

Proof: (I) Let x be any arbitrary element of $(A \cup B)'$,

then, $x \in (A \cup B)' \Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A \wedge x \notin B$

$\Rightarrow x \in A' \wedge x \in B'$

$\Rightarrow x \in A' \cap B'$.

$\therefore (A \cup B)' \subseteq A' \cap B'$ ——— (1)

Again, let y be an arbitrary element of $A' \cap B'$, then

$y \in A' \cap B' \Rightarrow y \in A' \wedge y \in B'$

$\Rightarrow y \notin A \wedge y \notin B$.

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)'$

$\therefore A' \cap B' \subseteq (A \cup B)'$ ——— (2)

From (1) and (2), we get

$(A \cup B)' = A' \cap B'$ Proved.

(II) Let x be an arbitrary element of $(A \cap B)'$,

then, $x \in (A \cap B)' \Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A' \vee x \in B'$.

$\Rightarrow x \in A' \cup B'$

$\therefore (A \cap B)' \subseteq A' \cup B'$ ——— (1)

Again, let y be an arbitrary element of $A' \cup B'$

Then, $y \in A' \cup B' \Rightarrow y \in A' \vee y \in B'$

$\Rightarrow y \notin A \vee y \notin B$

$\Rightarrow y \notin (A \cap B)$

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Rest part of Q1 (II)

$$\Rightarrow y \in (A \cap B)'$$

$$\therefore A' \cup B' \subseteq (A \cap B)' \text{ ——— } (2)$$

From (1) and (2)

$$(A \cap B)' = A' \cup B'$$

Proved.

Q. 2. De Morgan's Laws in another form

Prove that $A-(B \cup C) = (A-B) \cap (A-C)$

Proof: Let x be an arbitrary element of $A-(B \cup C)$, then

$$x \in A-(B \cup C) \Rightarrow x \in A \wedge x \notin (B \cup C)$$
$$\Rightarrow x \in A \wedge (x \notin B \wedge x \notin C)$$
$$\Rightarrow (x \in A \wedge x \notin B) \wedge (x \in A \wedge x \notin C)$$
$$\Rightarrow x \in (A-B) \wedge x \in (A-C)$$
$$\Rightarrow x \in (A-B) \cap (A-C)$$
$$\therefore A-(B \cup C) \subseteq (A-B) \cap (A-C) \text{ ——— } (1)$$

Again, let y be an arbitrary element of $(A-B) \cap (A-C)$.

Then, $y \in (A-B) \cap (A-C) \Rightarrow y \in (A-B) \wedge y \in (A-C)$

$$\Rightarrow (y \in A \wedge y \notin B) \wedge (y \in A \wedge y \notin C)$$
$$\Rightarrow y \in A \wedge (y \notin B \wedge y \notin C)$$
$$\Rightarrow y \in A \wedge y \notin (B \cup C)$$
$$\Rightarrow y \in A-(B \cup C)$$
$$\therefore (A-B) \cap (A-C) \subseteq A-(B \cup C) \text{ ——— } (2)$$

From (1) and (2), we get

$$A-(B \cup C) = (A-B) \cap (A-C)$$

Proved.

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